Holdups and overinvestment in capital markets

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Abstract

This paper considers a decentralized capital market characterized by trading frictions in which firms and suppliers need to make investment decisions before meeting with each other and bargaining over the price of capital. The resulting holdup problem provides firms with a strategic incentive to overaccumulate capital so as to reduce their marginal productivity and thus the bargained price. In equilibrium, this strategic incentive can outweigh the usual distortionary effects of holdup problems that on their own would lead to underinvestment, thus resulting in the economy to overinvest. In a setting with both capital and labor, the holdup problem in capital markets interacts with holdup problems in labor markets. This presents firms with a trade-off that has non-trivial equilibrium effects and that – depending on the substitutability of capital and labor and the firm’s bargaining power in each market – can mitigate or exacerbate the overinvestment result.

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1. Introduction

Factor specificity and contract incompleteness are pervasive phenomena in economic transactions and considered to be a major source of inefficiency. Whenever one party expends resources that increase the value of a productive relationship relative to outside options (i.e. specificity) and other parties can appropriate some of the rents arising from the investment (i.e. contract
incompleteness), a holdup problem arises. Holdup problems typically reduce the incentive to invest (e.g. Simons [50], Grout [30]) and, in equilibrium, lead to underutilization of resources, missing technology adoption, and excessive destruction (e.g. Caballero and Hammour [7]).

The present paper examines the consequences of holdup problems in decentralized capital markets characterized by trading frictions and shows that the forces usually associated with underinvestment – factor specificity and contract incompleteness – can lead to exactly the opposite equilibrium outcome: overinvestment. The result arises naturally in an environment in which firms and suppliers need to make investment decisions before meeting with each other and bargaining over the price. The trading frictions imply that the match between firms and suppliers generates surplus. Ex-post price negotiation implies that each party can appropriate part of this surplus. The resulting holdup problem distorts ex-ante investment decisions and provides firms with a strategic incentive to overaccumulate capital so as to reduce marginal productivity and thus the price of capital. In equilibrium, this strategic incentive can outweigh the usual distortionary effects of holdup problems that on their own would lead to underinvestment, thus resulting in the economy to overinvest.

The forces behind overinvestment are, from a mechanical point of view, the same as the ones leading to overhiring in labor markets with employment-at-will contracts, as analyzed originally by Stole and Zwiebel [52,53] and extended to a modern general equilibrium labor search context by Smith [51], Cahuc and Wasmer [9,10], and Cahuc, Marque and Wasmer [11]. The novelty of the present paper is to argue that similar frictions are relevant for capital markets and provide firms with a strategic incentive to overaccumulate capital. The paper then extends the analysis to a multifactor setting with holdup problems in different input markets and shows that the strategic reaction of firms has non-trivial equilibrium effects.

The notion that the allocation of capital from suppliers to firms is subject to trading frictions and ex-post bargaining contrasts with much of the investment literature in macroeconomics and finance, which focuses on different types of adjustment costs and credit constraints but maintains that the price of capital is determined competitively. Yet, a wide variety of capital – ranging from real assets such as structures and equipment to financial assets such as bank loans, fixed-income debt and derivatives – trade in decentralized or so-called ‘over-the-counter’ markets. Since much of this capital is specific in terms of quality, task or location, matching between firms and suppliers is likely to involve both material and opportunity costs. These costs give rise to match surplus that needs to be split somehow, thus opening the door to bargaining.

Recent empirical work suggests that for many decentralized capital markets, trading frictions and bargaining are not just a theoretical curiosity but quantitatively relevant. As a result, a burgeoning literature has emerged that attempts to explain the different phenomena with search-

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1 Also see Wolinsky [59] for a dynamic extension of Stole and Zwiebel’s [52] partial equilibrium analysis.
2 See Duffie, Garleanu and Pedersen [19,20] for examples of decentralized financial asset markets that are subject to trading frictions and bargaining. See Rauch [48] and Nunn [44] for a list of real asset markets without organized exchange nor reference prices in trade publications.
3 See Pulvino [45] and Gavazza [25] for the quantitative relevance of trading frictions and bargaining in commercial aircraft markets – presumably one of the most homogenous and frictionless real asset markets – as well as Ramey and Shapiro [46,47], Maksimovic and Phillips [42], Eisfeldt and Rampini [23] or Kurmann and Petrosky-Nadeau [36] for evidence on trading frictions and bargaining in real asset markets in general. For quantitative evidence of trading frictions and bargaining in over-the-counter financial markets, see Dell’Ariccia and Garibaldi [14], Green, Li and Schuerhoff [28] or Afonso and Lagos [2] among others.
The present paper shares with this literature the premise that search and bargaining are important features of many capital markets but focuses on the normative question of holdups and inefficient investment. This focus is motivated by the observation that investment decisions often require lengthy planning during which market research is conducted, financing is arranged, and permits are obtained. These *time-to-plan* constraints can be substantial and have been used widely in both the real options and the business cycle literature. But the same constraints also imply that firms and capital suppliers must make important investment decisions *before* they meet and bargain over the price. The sequential nature of this process captures the essence of what is meant here by ex-post bargaining and what prevents firms and suppliers from committing ex-ante to binding arrangements or, equivalently, bargain simultaneously over both price and quantity, that would circumvent holdup problems. This seems especially relevant for capital rentals where lease terms are open for renegotiation but the amount of capital rented remains fixed during the life of the lease. The maintained assumption of the paper is therefore that contracts are non-binding or at least not binding for very long.

The paper proceeds as follows. Section 2 derives the overinvestment result in a basic dynamic general equilibrium model with random search and ex-post bargaining. Apart from a modification to endogenize the supply of capital, the model mirrors the labor search environment that Smith [51], Cahuc and Wasmer [9,10], and Cahuc, Marque and Wasmer [11] use to analyze overhiring in labor markets with employment-at-will contracts. The random search assumption is appealing because of its empirical relevance and its convenience for equilibrium analysis. At the same time, the firm’s strategic incentive to overaccumulate capital emerges independently of the search friction as long as one maintains that trading frictions prevent firms from costlessly replacing capital from a given supplier with equivalent capital from another supplier. Furthermore, while the model resembles in many ways the neoclassical growth benchmark, capital should be understood broadly and include not only physical capital but also other types of intermediate inputs and financial assets traded in decentralized markets.

Ex-post bargaining is modeled as in Stole and Zwiebel [52] and captures the idea that due to the above discussed complexities of investment, firms and workers cannot commit to ex-ante binding contracts. At any time before production starts, a matched supplier may enter into pairwise renegotiation with the firm. Likewise, a firm may call in any of its suppliers for renegotiation. Within each pairwise negotiation, the firm and the supplier play an alternating-offer game with exogenous probability of breakdown as in Binmore, Rubinstein and Wolinsky [5].

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4 Prominent examples for financial asset markets are Wasmer and Weil [56], Duffie, Garleanu and Pedersen [19,20], Lagos and Rocheteau [38,40], Vayanos and Weil [55], and Afonso and Lagos [3] among many others. Also see Williamson and Wright [58] for a general review of search-theoretic models for asset pricing. For real asset markets, relevant examples include Kurmann and Petrosky-Nadeau [36] or Gavazza [26].

5 See Dixit and Pindyck [18] for a review of the importance of time-to-plan for the real options literature; and Christiano and Todd [13] or Edge [22] for applications of time-to-plan in business cycle models. Time-to-plan in these literatures is motivated by empirical evidence in Mayer [43], Jorgenson and Stephenson [33], Kainer [35] among others who find that investment lags behind its determinants by 6 to 12 quarters and that the initial planning phase requires little resource costs relative to the actual investment outlay.

6 The idea that firms need to decide on projects prior to the actual investment is similar in spirit to the analysis of innovation and ‘the market for ideas’ by Silveira and Wright [49].

7 Examples in the literature of such ex-ante binding arrangements are long-term contracts (e.g. Williamson [57]); reallocation of property rights (e.g. Klein, Crawford and Alchian [34], Grossman and Hart [29], Hart and Moore [31]); punishment schemes (e.g. MacLeod and Malcomson [41]); or price posting with commitment (Acemoglu and Shimer [1]). See Caballero [6] for a critique about the realism of such arrangements.
The resulting unique subgame-perfect price is identical for all suppliers and solves the generalized Nash bargaining problem over the marginal match surplus. This solution depends on the firm’s infra-marginal productivities because the involved parties understand that if negotiations with one supplier break down, the price with all other suppliers is renegotiated based on the new marginal match surplus. As long as technology exhibits decreasing returns, the firm strategically reacts to this situation by overaccumulating capital so as to reduce marginal productivity and thus the ex-post negotiated price. For suppliers, no such strategic incentive arises because at the time of the capital production decision, suppliers do not know the identity of the firms with which they match and therefore consider a given firm’s capital stock as exogenous.\(^8\)

In equilibrium, the firm’s strategic incentive interacts with the usual distortionary effects of holdup problems that on their own would lead to underinvestment. The resulting allocation is always inefficient; and the aggregate investment level exhibits an inverted U-shape in the supplier’s relative bargaining power, with a range of bargaining powers associated with equilibrium outcomes for which the firm’s strategic overinvestment incentive outweighs the typical underinvestment effect of holdups.

Section 3 extends the analysis to a multifactor setting. First, the paper assesses the robustness of the firm’s strategic overinvestment incentive to a second capital factor that is subject to the same trading frictions and ex-post bargaining. Second, the paper introduces labor alongside capital to analyze how the holdup problem in the capital market interacts with a standard holdup problem arising from labor search and employment-at-will contracts that, on its own, would lead to underinvestment and overemployment. In both cases, a double holdup problem arises that mitigates (exacerbates) the firm’s strategic incentive to overaccumulate capital if the two factors are complements (substitutes) because a higher capital stock increases (decreases) marginal productivity of the other factor, thus worsening (attenuating) the firm’s holdup problem in the other factor market.

In equilibrium, the double holdup problem has non-trivial effects that depend on the firm’s production function and its bargaining power in the two factor markets. For the special case in which the production function is constant-returns-to-scale and the firm’s bargaining power is equal across markets, the strategic incentive to overaccumulate capital in one market is exactly offset by the incentive to overaccumulate capital/overhire labor in the other market. If, in addition, the bargaining power in each market achieves efficient market tightness (i.e. Hosios’ condition), equilibrium allocations are efficient because the usual distortionary effects of holdup problems exactly reflect the externalities from the search frictions. This knife-edge case illustrates that multiple holdup problems do not necessarily amplify each other as is often the case in the literature (e.g. Aruoba, Waller and Wright [4]).

More generally, however, equilibrium allocations in the multifactor setting remain inefficient and there continues to exist a range of bargaining powers for which the economy overinvests. In particular, if production is decreasing-returns-to-scale and the firm has equal bargaining powers in both markets, the firm’s incentive to overaccumulate capital persists independent of whether the two factors are complements or substitutes. This incentive can easily outweigh the usual distortionary effects of holdup problems that, on their own, would lead to underinvestment.

Section 4 concludes by discussing policy implications, and why the firm’s strategic overinvestment incentive analyzed in this paper may offer interesting explanations for a number of empirical observations.

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\(^8\) In other words, the firm has the right-to-manage its capital stock. This is discussed further in Section 2.
2. Overinvestment in a basic model

The basic model is intentionally kept simple to convey intuition. Capital is the only factor of production and its allocation from suppliers to firms is governed by a random search matching function. Once new matches have occurred, firms and suppliers – whether newly matched or incumbent – bargain over the price of capital. In other words, capital is rented with suppliers retaining the residual property right. This renting assumption is motivated by the empirical observation that a substantial fraction of both financial and real assets are not purchased outright but leased. In addition, the assumption that capital is purchased (or, more generally, that rental rates of different capital units are not all (re-)negotiated at the same time) would introduce an additional strategic motive for the firm to overinvest. Further details are provided at the end of this section.

Apart from a modification to endogenize the supply of capital, the model mirrors the labor search environment that Smith [51], Cahuc and Wasmer [9,10] and Cahuc, Marque and Wasmer [11] use to study overemployment. Derivations are therefore relegated to Appendix A.

2.1. Environment

The economy is populated by a continuum of infinitely-lived atomistic firms \( \iota \in [0,1] \) and capital suppliers \( \omega \in [0,1] \). Time is discrete and discounted at rate \( \beta = (1 + r)^{-1} \). Both firms and capital suppliers derive linear utility from consuming a numeraire good that firms produce with capital stock \( k \) using technology \( f(k) \). This technology is twice differentiable, strictly increasing and concave in \( k \) and satisfies the usual Inada conditions.

The allocation of capital from suppliers to firms occurs in two stages. In the first stage, firms post ‘ventures’ \( v \) at flow cost \( \gamma \) to search for new capital while capital suppliers transform numeraire into ‘liquid’ capital \( l \) to search for firms with open ventures. There is no resource cost for this transformation. However, capital suppliers bear an irreversible opportunity cost because committing liquid capital to the market reduces current consumption independent of whether the liquid capital is matched with a firm or not. The allocation of liquid capital to ventures is governed by a random-search matching function \( m(V,L) \leq \min(V,L) \), where \( V = \int_0^1 v d\iota \) and \( L = \int_0^1 l d\omega \) denote the total mass of new ventures and liquid capital, respectively. For simplicity, this function is assumed to exhibit constant-returns-to-scale with \( \lim_{V \to 0} m_V(\theta) = 1 \) and \( \lim_{V \to \infty} m_V(\theta) = 0 \), where \( m_V(\theta) \equiv \frac{\partial m(V,L)}{\partial V} \theta \) and \( \theta \equiv V/L \) denotes capital market tightness. Accordingly, each venture matches with a unit of liquid capital with probability

\[ m(V,L) = \frac{VL}{(LV + VX)^{1/\chi}} \]
\( p(\theta) \equiv m(V, L)/V \) and each unit of liquid capital matches with a venture with probability \( q(\theta) \equiv m(V, L)/L \).

In the second stage, newly matched capital joins the firm’s existing capital stock net of depreciation to form the firm’s new capital stock. Firms then produce and open new ventures while capital suppliers combine unmatched liquid capital with numeraire to search for new ventures.

Given these assumptions, the evolution of a firm’s productive capital stock is described by
\[
k' = (1 - \delta)k + p(\theta)v, \tag{1}
\]
where \( \delta \) denotes the exogenously given rate of depreciation. Likewise, the evolution of a capital supplier’s matched capital rented out to firms is described by
\[
\kappa' = (1 - \delta)\kappa + q(\theta)l, \tag{2}
\]
where \( \kappa \) is the matched capital stock of the capital supplier. Finally, unmatched capital returning to the capital supplier in the beginning of next period is defined as
\[
u' = (1 - q(\theta))l. \tag{3}
\]
The temporal distinction in these equations highlights the sequential nature of the investment process: first, firms open ventures and capital suppliers provide liquid capital; second, ventures match with liquid capital and unmatched liquid capital returns to the suppliers. Also notice that while the environment mostly resembles one of physical capital accumulation, capital can be easily interpreted as an intermediate good (with the depreciation rate taking the value \( \delta = 1 \)) or a financial asset. In the latter case, \( \delta \) would be interpreted as a separation rate; and Eq. (3) would be modified to accommodate returns of separated capital. More generally, the model could be easily extended to allow for separation and reallocation of used capital.

2.2. Efficient allocation

The planner’s problem that achieves the (constrained) efficient allocation is
\[
P(K, U) = \max_{L, V} \left[ f(K) - \gamma V - (L - U) + \beta P(K', U') \right]
\]
subject to
\[
k' = (1 - \delta)k + m(V, L),
\]
\[
u' = L - m(V, L),
\]
where \( K = \int_0^1 k dt = \int_0^1 \kappa d\omega \) and \( U = \int_0^1 u d\omega \) denote the aggregate stocks of matched and unmatched capital, respectively. The term \( f(K) - \gamma V - (L - U) \) is the current-period aggregate

with \( \chi > 1 \). This formulation has been used by Den Haan, Ramey and Watson [17] and many others in the labor search literature. In the present context, the denominator \( (L^\chi + V^\chi)^{1/\chi} \) can be interpreted as the number of submarkets in which the capital market is segmented. A match occurs when a liquid capital unit and a venture are in the same submarket. The other ventures and liquid capital units remain unmatched. Hence, if liquid capital \( L \) and ventures \( V \) are assigned randomly to submarkets (due, for example, to information imperfections about potential suppliers and firms), then the total number of matches is \( VL/(L^\chi + V^\chi)^{1/\chi} \).

12 Since firms and capital suppliers are atomistic, the law of large numbers applies such that the expected number of matches equals the realized number of matches. Also note that the match probabilities \( q(\theta) \) and \( p(\theta) \) reflect equilibrium outcomes; i.e. during the bargaining stage, firms and suppliers have the option to break off negotiations but in equilibrium, this would be suboptimal.
utility from numeraire consumption, consisting of aggregate output by firms minus resources expended for opening new ventures and committing net new liquid capital to the market.\footnote{13}

The analysis focuses exclusively on steady state equilibria where $X = X'$ for all variables. Appendix A.1 provides an explicit derivation of the planner’s problem. Here, a simple characterization of the resulting equilibrium allocation is given.

**Proposition 1.** There exists a unique efficient allocation $\{K^P, \theta^P\}$ that solves

\[
\gamma = m_V(\theta) \left[ \frac{f_K(K)}{r + \delta} - \beta \right]
\]

\[
1 - \beta = m_L(\theta) \left[ \frac{f_K(K)}{r + \delta} - \beta \right]
\]

**Proof.** Appendix A.1. □

Eq. (4) says that the cost of an additional venture equals the marginal increase in matches with respect to ventures times the discounted net surplus of an additional match. Eq. (5) says that the opportunity cost of an additional unit of liquid capital equals the marginal increase in matches with respect to liquid capital times the discounted net surplus of an additional match. Combining the two equations to eliminate the marginal net surplus yields the following implicit solution for the efficient level of capital market tightness

\[
\theta^P = \frac{1 - \beta}{\gamma} \frac{1 - \epsilon(\theta^P)}{\epsilon(\theta^P)}
\]

where $\epsilon(\theta) \equiv m_L(\theta)/q(\theta)$ and $1 - \epsilon(\theta) \equiv m_V(\theta)/p(\theta)$ denote the matching elasticities with respect to liquid capital and ventures, respectively. Given $\theta^P$, either (4) or (5) then pins down the efficient matched capital stock $K^P$; and the efficient levels of liquid capital $L^P$, ventures $V^P$ and unmatched capital $U^P$ are computed from the two constraints and the definition of capital market tightness.

2.3. Decentralized allocation

In the decentralized economy, the surplus from matching is split via rental rate $\rho$. Following Stole and Zwiebel [52], a matched supplier may enter into pairwise negotiation with the firm at any time before production starts. Likewise, a firm may call in any matched supplier for negotiation. As discussed in the introduction, this assumption of continuous pairwise negotiation captures the idea that due to various complexities of the investment process, contracts are non-binding or at least not binding for very long.

Stole and Zwiebel [52] formally show that this negotiation protocol implies that each supplier (whether newly matched or incumbent) is the marginal supplier and negotiates an identical rental rate over the marginal match surplus. Since this marginal match surplus is a function of the firm’s capital stock, firms consider the rental rate as endogenous in their venture posting decision; i.e.

\footnote{13 As mentioned above, this definition of consumption makes clear that while transforming numeraire into liquid capital can be done at no resource cost, there is an opportunity cost for liquid capital because it reduces current consumption. This is analogous to investment in a frictionless neoclassical model only that now, aggregate net investment by capital suppliers is defined as $L - U$.}
\( \rho = \rho(k) \). For suppliers, by contrast, no such strategic incentive arises because at the time of the liquid capital decision, suppliers do not know the identity of the firms with which they match and therefore consider a given firm’s capital stock as exogenous.

Given these assumptions, the problem of a firm with matched capital stock \( k \) is

\[
J(k) = \max_v \left[ f(k) - \rho(k)k - \gamma v + \beta J(k') \right]
\]

subject to (1); and the problem of a supplier with matched capital \( \kappa \) and unmatched liquid capital \( u \) is

\[
V(\kappa, u) = \max_l \left[ \rho \kappa - l + u + \beta V(\kappa', u') \right]
\]

subject to (2) and (3). In steady state, the firm’s problem leads to the following demand for capital

\[
\rho(k) = f_k(k) - \rho_k(k)k - (r + \delta) \frac{\gamma}{\rho(\theta)}. \tag{7}
\]

The term \( \rho_k(k)k \) captures the firm’s strategic incentive resulting from the holdup problem and is described below. Without this term, (7) would reduce to a standard capital demand condition, stating that the firm’s marginal product of capital equals the rental rate plus investment adjustment cost (here, the average venture posting cost per unit of capital). In turn, the supplier’s problem implies the following steady state condition for new liquid capital

\[
1 = q(\theta) \frac{\rho}{r + \delta} + (1 - q(\theta)) \beta. \tag{8}
\]

Committing one unit of liquid capital costs one unit of numeraire. With probability \( q(\theta) \), the liquid capital gets matched with a firm yielding a discounted present-value of \( \rho / (r + \delta) \). With probability \( 1 - q(\theta) \), the liquid capital remains unmatched and returns to the supplier in the beginning of next period, implying a discounted value of \( \beta \).

Eqs. (7) and (8) can be used to discuss the distortionary effects of holdup problems usually emphasized in the literature. Suppose that the share of the surplus captured by the supplier decreases, leading to a fall in the rental rate \( \rho \). Everything else constant, this reduces the holdup problem for the firm, thereby increasing its incentive to post ventures. Vice versa, the holdup problem for the supplier becomes more severe, thereby decreasing its incentive to provide liquid capital. As will be shown below, these two opposing forces typically do not cancel each other out in equilibrium and would, on their own, lead to underinvestment.

As described above, Stole and Zwiebel’s [52] assumption of pairwise continuous bargaining implies that the rental rate is a function of the marginal match surplus, defined as \( J_k(k) + V_\kappa(\kappa, u) - V_u(\kappa, u) \) where \( J_k(k) \), \( V_\kappa(\kappa, u) \) and \( V_u(\kappa, u) \) denote the different marginal values of the firm and the supplier (see Appendix A.2 for formal definitions). Following Stole and Zwiebel [52] one more step, if within each pairwise negotiation, the firm and the supplier play an alternating-offer game with exogenous probability of breakdown as in Binmore, Rubinstein and Wolinsky [5], then the unique subgame-perfect equilibrium solves the generalized Nash bargaining problem over the marginal match surplus; i.e. the renegotiation-proof rental rate with each of the suppliers is such that \( \phi J_k(k) = (1 - \phi)(V_\kappa(\kappa, u) - V_u(\kappa, u)) \), where \( \phi \) denotes the bargaining power of capital suppliers. In steady state, this rental rate can be expressed as

\[
\rho(k) = \phi \left[ f_k(k) - \rho_k(k)k \right] + (1 - \phi) \frac{r + \delta}{1 + r}. \tag{9}
\]

The rental rate is a weighted average of the firm’s and the supplier’s respective outside option (which together define marginal surplus), with the weights depending on the relative bargaining
power of the two parties. The supplier’s outside option is to have an additional unmatched unit of capital in the beginning of next period, which has a discounted annuity value of \((r + \delta)/(1 + r)\).

The firm’s outside option is to produce with one marginal unit of capital less, which reduces production by \(-\rho_k(k)k\) because all remaining suppliers come in for renegotiation. This effect is a direct consequence of the trading friction preventing firm’s from costlessly replacing capital from a supplier with whom negotiations broke down with equivalent capital from another supplier.

To derive an explicit expression for \(\rho_k(k)\), notice that (9) is a non-homogenous linear differential equation of \(\rho\) in \(k\) with solution

\[
\rho(k) = k^{-\frac{1}{\delta}} \int_0^k \frac{1-\phi}{z^{1+\phi}} f_{z}(z) dz + (1-\phi) \frac{r + \delta}{1+r}.
\]

The rental rate is a function of the contribution of each unit of capital to the firm’s production. Intuitively, this dependence on infra-marginal productivities arises because the change in rental rate if negotiations with one supplier break down depends itself on what would happen to the rental rate if there was a hypothetical breakdown with one of the remaining capital suppliers – a recursive argument that can be carried through all the way to the first capital supplier (see Stole and Zwiebel [52] for a demonstration). Given (10), the firm’s capital demand in (7) can be expressed as

\[
\rho(k) = OI(k) \times f_k(k) - (r + \delta) \frac{\gamma}{p(\theta)}
\]

where

\[
OI(k) \equiv 1 - \frac{k^{-\frac{1}{\delta}} \int_0^k \frac{1-\phi}{z^{1+\phi}} f_{zz}(z) dz}{f_k(k)}
\]

is defined as the firm’s ‘overinvestment factor’, analogous to the definition of the overemployment factor in Cahuc, Marque and Wasmer [11]. This overinvestment factor captures the essence of the firm’s strategic response to the holdup problem and has the following properties.

Proposition 2. For \(\phi > 0\) and \(f_{zz}(z) \leq 0\) with strict inequality over some segment of \(z \in [0,k]\), \(OI(k) > 1\) and \(\partial OI(k)/\partial \phi > 0\).


\(OI(k) > 1\) together with the fact that \(OI(k) \times f_k(k)\) is decreasing in \(k\) (see Appendix A.2) implies that the firm’s optimal new ventures exceed the level warranted by its marginal productivity. Intuitively, by increasing ventures and therefore the capital stock, the firm drives down productivity and with it the rental rate, thus reducing the match surplus that each supplier can appropriate. \(\partial OI(k)/\partial \phi > 0\), in turn, implies that the larger the share of the surplus the supplier can appropriate, the larger the firm’s incentive to overaccumulate capital.

To examine the equilibrium effects of the firm’s strategic incentive and how it interacts with the usual distortionary effects of the holdup problem, combine the rental rate in (9) with the firm’s capital demand in (11) to obtain

\[
\frac{\gamma}{p(\theta)} = (1-\phi) \left[ \frac{OI(k) \times f_k(k)}{r + \delta} - \beta \right].
\]
This equation implies that the firm’s average cost per match equals its share of the match surplus. Likewise, supply condition (8) can be combined with the rental rate in (9) to obtain
\[ 1 - \beta q(\theta) = \phi \left[ \frac{O(I(k) \times f_k(k))}{r + \delta} - \beta \right]. \tag{14} \]
This equation implies that in equilibrium, the capital supplier’s average opportunity cost per match equals its share of the match surplus. After aggregation over all firms and suppliers, the two equations lead to the following characterization of the decentralized allocation.

**Proposition 3.** There exists a unique decentralized allocation \( (K^D, \theta^D) \) that solves (13) and (14). This allocation is always inefficient. In particular, for \( \phi = \epsilon(\theta^P) \), \( \theta^D = \theta^P \) and \( K^D > K^P \).

**Proof.** Appendix A.2. □

To provide intuition for this proposition, combine (13) with (14) to eliminate the bracketed expression. This yields a unique solution for capital market tightness
\[ \theta^B = \frac{1 - \beta}{\gamma} \frac{1 - \phi}{\phi}, \tag{15} \]
which closely resembles the efficient solution in (6), with the difference that \( \phi \) replaces the match elasticity \( \epsilon(\theta) \). For \( \phi = \epsilon(\theta^P) \), the bargaining power is just strong enough to exactly reflect the externality from adding another venture, respectively another unit of liquid capital to the market. Hence, capital market tightness is efficient; i.e. \( \theta^B = \theta^P \). This is the equivalent of Hosios’ [32] famous efficiency condition in labor search models with bargaining. Comparing (13) with (14) with the social planner counterparts in (4) and (5), it is then straightforward to see that the decentralized allocation must be inefficient because \( O(I(K)) > 1 \) for any \( K \). Since \( O(I(K)) \times f_K(K) \) is decreasing in \( K \) by Proposition 2, this implies that \( K^D > K^P \) at \( \phi = \epsilon(\theta^P) \); i.e. the economy overinvests.

For \( \phi < \epsilon(\theta^P) \), the share of the surplus appropriated by the supplier is smaller, thus worsening the supplier’s holdup problem. Vice versa, the firm’s holdup problem is less severe, which in turn reduces the firm’s strategic incentive to overaccumulate capital (i.e. by Proposition 2, \( O(I(K)) \) is increasing in \( \phi \)). In equilibrium, these opposing forces result in inefficiently high capital market tightness; i.e. \( \theta^B > \theta^P \); as can be seen by comparing (6) with (15). Furthermore, it is possible to show (see Appendix A.2) that the equilibrium effect of lowering \( \phi \) below \( \epsilon(\theta^P) \) is a decrease in \( K^D \) and that for sufficiently low values of \( \phi \), the equilibrium capital stock falls below the efficient level with \( \lim_{\phi \to 0} K^D = 0 \). Conversely, for \( \phi > \epsilon(\theta^P) \), \( \theta^B < \theta^P \) and \( K^D \) locally increases even further above \( K^P \) before eventually decreasing and falling below the efficient level with \( \lim_{\phi \to 1} K^D = 0 \). In other words, \( K^D \) follows an inverted U-shape in \( \phi \), with an entire range of values of \( \phi \) between 0 and 1 for which \( K^D > K^P \). This is depicted in Fig. 1.

For illustrative purposes, the figure also displays the equilibrium capital stock under the counterfactual assumption that the firm has no strategic incentive to overaccumulate capital (i.e. \( O(I(K)) \) in Eqs. (13) and (14) is exogenously set to 1). In this case, the only forces affecting the

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14 As \( \phi \to 0 \), the usual distortionary effect of the holdup problem for the supplier becomes so strong that its provision of liquid capital goes to zero; i.e. \( L = 0 \). Vice versa, as \( \phi \to 1 \), the usual distortionary effect of the holdup problem for the firm becomes so strong that its venture postings go to zero; i.e. \( V = 0 \). In both cases the market breaks down and there is no investment.
equilibrium are the usual distortionary effects of holdup problems. Except for $\phi = \epsilon(\theta^P)$, these effects do not cancel each other out and would, on their own, lead to underinvestment.¹⁵ This illustrates that the firm’s strategic overinvestment incentive can easily outweigh the usual distortionary effects of holdup problems emphasized in the literature, thus resulting in the economy to overinvest.

### 2.4. Behind overinvestment

The two main ingredients for the holdup problem to arise are trading frictions in the allocation of capital and absence of ex-ante binding contracts. While both of these ingredients receive ample empirical support, as emphasized in the introduction, it remains instructive to discuss the underlying assumptions in more detail.

Trading frictions are implemented in the model through search and matching. While the formulation of trading frictions is convenient for equilibrium analysis, the same holdup problem and overinvestment result would arise in alternative environments as long as one maintains that firms and sellers need to make ex-ante decisions about new ventures and liquid capital provision, respectively; i.e. once matched, firms and suppliers cannot negotiate simultaneously over price and quantity. For the same reason, firms cannot costlessly replace capital from one supplier with equivalent capital from another supplier in the event negotiations with a supplier break down.¹⁶

¹⁵ The underinvestment result when abstracting from the firm’s strategic incentive is quite different from the implications of the basic labor search model where a worker’s bargaining power below the elasticity of the matching function is associated with inefficiently high equilibrium employment (see Cahuc and Wasmer [10]). The reason for this difference is that in the basic search model, labor force participation (respectively search intensity by workers) is fixed.

¹⁶ Equivalently, the holdup problem and the overinvestment result would arise if firms were able to obtain new capital from already matched suppliers as long as this new capital cannot be delivered instantaneously (otherwise, there would be costless replacement).
As discussed in the introduction, these assumptions seem to be relevant in many situations due to time-to-plan constraints and the specific nature of capital.

Absence of ex-ante binding contracts is implemented in the model through the assumption of continuous pairwise bargaining between firms and suppliers after the match has occurred. However, both the holdup problem and the overinvestment result would obtain for alternative bargaining protocols as long as each party can appropriate some of the match surplus ex-post. Furthermore, as Stole and Zwiebel [52] demonstrate, the solution of the continuous pairwise bargaining game is equivalent to the Shapley values of a corresponding cooperative game. Hence, the absence of cooperation among suppliers is by itself not important. What is important, however, is that suppliers cannot form a perfect coalition ex-ante and consider the firm’s capital stock as an endogenous variable when making liquid capital decisions (or alternatively, that a single supplier provides all of the firm’s capital stock). Such ex-ante coalition-building is not a general feature of capital markets, presumably because at the time of the supplier’s liquid capital decision, the supplier is unlikely to know the identity of the firms with which it will match and therefore is unlikely to know the other suppliers of a given firm. In contrast, the firm by its very nature knows its capital stock and can manipulate it to obtain a strategic advantage in future lease negotiations.17

Aside from trading frictions and absence of ex-ante binding contracts, the model makes a number of other assumptions that are worthwhile examining. Consider first how results would change if, upon matching, firms purchased rather than rented capital from suppliers; i.e. if there was a reallocation of property rights (e.g. Klein, Crawford and Alchian [34], Grossman and Hart [29], Hart and Moore [31]) or, equivalently, an ex-post binding long-term contract. As shown in an earlier version of the paper, as long as this price negotiation occurs ex-post, the holdup problem and the overinvestment result would still emerge except that the firm would only bargain with newly matched suppliers and thus, the resulting price next period would be a function of infra-marginal productivities between \( k' \) (the capital stock if negotiations with all new suppliers are successful) and \( (1 - \delta)k \) (the capital stock in case negotiations with all new suppliers break down). In addition, as formally shown by Kurmann and Rabinovich [37], the purchasing assumption would provide the firm with a second intertemporal strategic incentive to overaccumulate capital because \( (1 - \delta)k \) (the lowest possible capital stock if all negotiations break down) and therefore the price depends on capital accumulation in previous periods. Hence, the purchasing assumption would reinforce the overinvestment result.

Second, consider the assumption that capital is perfectly divisible and homogenous. In reality, capital projects are often indivisible and heterogenous. Stole and Zwiebel [52] derive their over-hiring result in a discretized environment and thus, factor indivisibilities per se are not a problem. The only thing that would change is that \( k \) and therefore \( \rho(k) \) would no longer be continuous, thus unnecessarily complicating the equilibrium analysis.18 Heterogeneity in capital inputs is

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17 Even if a (coalition of) supplier(s) found it optimal to strategically withhold capital from a firm so as to drive up the firm’s marginal productivity and therefore the bargained price, the firm could ex-ante post more ventures so as to match with additional suppliers and attain its desired capital stock. In other words, the firm naturally has the right-to-manage its desired capital stock in this environment.

18 It is also possible to show that the assumption of atomistic suppliers represents a limiting case that occurs when the search cost per supplier is zero. If the search cost per supplier was positive instead, the firm would optimally obtain equal amounts of capital from a finite number of suppliers. This would make the firm’s incentive to overaccumulate capital worse.
more difficult to analyze and is therefore treated in detail in the next section for an environment with two capital inputs.

Third, consider the assumption of decreasing returns to capital. If $f_k(k)$ was constant, the firm would have no incentive to overinvest because the marginal surplus of a match and therefore the rental rate would not be a function of capital. By the same argument, if firms produced with constant-returns-to-scale technology in capital and another factor (e.g. labor) that is hired ex-post in a competitive spot market, the overinvestment result would disappear. This is because with constant-returns-to-scale technology, the marginal productivity of capital is proportional to the price of the other factor, which, in a competitive spot market, is taken as exogenous by the firm.\footnote{See Cahuc and Wasmer \cite{9} who show this result in a labor search context.} The incentive to overinvest reappears, however, under the assumption that the firm’s optimal decision for the other factor occurs before the price of capital is negotiated. If, in addition, this other factor is also subject to holdup problems, then the firm’s strategic behavior in each market has non-trivial general equilibrium effects. These effects are analyzed in detail in the next section.

The assumption of decreasing returns technology in capital is one of the key differences to Caballero and Hammour \cite{7} who analyze the equilibrium implications of holdup problems and conclude that there is generally underinvestment rather than overinvestment. In their model, production is exogenously given and thus, the concept of the firm as an independent optimizing entity open to strategic investment incentives is absent. This comparison highlights the crucial role of microeconomic structure for the analysis of holdup problems and shows that an arguably realistic change in assumptions – the presence of independently optimizing firms with diminishing returns to capital – can overturn the usual underinvestment result of holdup problems emphasized by the literature.

3. Overinvestment with multiple factors of production

This section extends the analysis to an economy with multiple factors of production in which each factor market is subject to search frictions and ex-post bargaining. The partial equilibrium analysis of the resulting multiple holdup problem borrows from Cahuc, Marque and Wasmer \cite{11} who, in the context of a labor search model, use spherical coordinate techniques to derive a closed-form expression for wages. As shown in their paper, existence and uniqueness of an equilibrium is impossible to establish for general functional forms in such a multi-factor setting. This section therefore focuses on two examples that are of particular interest for the present analysis and for which existence of a unique equilibrium allocation can be established.

The first example is a decreasing-returns-to-scale production function in two capital inputs. The main objective of this example is to assess the robustness of the overinvestment result to the introduction of a second capital input that is either a substitute or a complement in the production function. The second example is a constant-returns-to-scale production function in capital and labor. The focus of this example is to analyze how the holdup problem in the capital market interacts with a standard holdup problem in the labor market that on its own leads to underinvestment and overemployment. Notice that all the results derived in the two-capital example carry through to a variant of the capital–labor example with decreasing-returns-to-scale. This variant is therefore not analyzed separately.
3.1. Two capital factors

The environment is identical to the basic model except that firms now produce with technology \( f(k_1, k_2) \), where \( k_1 \) and \( k_2 \) denote two different capital factors that are rented from two separate \([0, 1]\) continua of capital suppliers. This technology is strictly increasing and concave in each of its arguments, satisfies the usual Inada conditions, and exhibits decreasing-returns-to-scale. Each capital market is subject to allocation frictions characterized by matching functions \( m(V_1, L_1) \), \( m(V_2, L_2) \) and market tightness \( \theta_1 = V_1/L_1, \theta_2 = V_2/L_2 \), respectively.\(^{20}\)

3.1.1. Efficient allocation

The planner’s problem that achieves the (constrained) efficient allocation is

\[
P(K_1, U_1, K_2, U_2) = \max_{L_1, V_1, K_2, V_2} \left[ f(K_1, K_2) - \gamma_1 V_1 - \gamma_2 V_2 - (L_1 - U_1) - (L_2 - U_2) + \beta P(K_1', U_1', K_2', U_2') \right]
\]

s.t. \( K_i' = (1 - \delta_i) K_i + m(V_i, L_i) \)

\( U_i' = L_i - m(V_i, L_i) \),

where \( K_i = \int_0^1 k_i \, dt = \int_0^1 k_i \, d\omega_i \), \( i = 1, 2 \) and so forth for the other aggregate quantities. As shown in Appendix B.1, for general production functions, the solution to this problem does not necessarily exist and, if it exists, is not necessarily unique. Given the decreasing-return-to-scale assumption, however, the following characterization of the equilibrium obtains.\(^{21}\)

**Proposition 4.** For homogenous production functions of degree \( d < 1 \) with

\[
f_{K_2}(K_1, K_2) f_{K_1 K_1}(K_1, K_2) - f_{K_1}(K_1, K_2) f_{K_1 K_2}(K_1, K_2) < 0 \quad \forall (K_1, K_2),
\]

there exists a unique efficient allocation \( \{K^*_i, \theta^*_P_i\}_{i=1}^2 \) that solves

\[
\theta_i = \frac{1 - \beta}{\gamma_i} \frac{1 - \epsilon(\theta_i)}{\epsilon(\theta_i)} \quad (17)
\]

\[
A^*_i \left( \theta^*_P \right) \equiv (r + \delta_i) \left( \frac{\gamma_i}{p(\theta_i)(1 - \epsilon(\theta_i))} + \beta \right) = K_2^{d-1} f_i(R, 1) \quad (18)
\]

where \( R \equiv K_1/K_2 \) and \( f_i(R, 1) \) denotes the partial derivative of \( f(R, 1) \) to its \( i \)-th argument.

**Proof.** Appendix B.1. \( \Box \)

Condition (17) is equivalent to (6) in the basic model and uniquely pins down tightness in each of the capital markets. Given \( \theta^*_P \) and \( \theta^*_P \), condition (18) is equivalent to (4) and uniquely defines \( R^P \), \( K^P_1 \) and \( K^P_2 \) as long as the two capital inputs are not too substitutable with each other (i.e. \( f_{K_1 K_2}(K_1, K_2) \) is either positive or not too negative so as to satisfy condition (16)).

\(^{20}\) To save on notation, the two matching functions and the corresponding match probabilities are described by the same functional form. Technically, this means that the trading frictions in the two markets are identical. Nothing about the results would change if different functional forms were used instead so as to allow for different trading frictions in each market.

\(^{21}\) The assumption of decreasing-returns-to-scale is in itself not necessary except that the present environment is too linear in other dimensions for there to be a determinate solution if production was constant-returns-to-scale.
As Appendix B.1 shows, a straightforward functional form for which this condition is always satisfied is the CES case \( f(k_1, k_2) = [k_1^\sigma + k_2^\sigma]^{1/\sigma} \) where \( 0 < \sigma < 1 \) is the elasticity of substitution. The efficient equilibrium values of the other aggregates are then computed from the different constraints and the definition of capital market tightness.

### 3.1.2. Decentralized allocation

Analogous to the basic model, the rental rate between firms and capital suppliers in each market is determined ex-post through continuous pairwise bargaining over the marginal match surplus using the alternating-offer protocol of Binmore, Rubinstein and Wolinsky [5]. As long as \( k_1 \) and \( k_2 \) are not perfect substitutes, the resulting rental rates are a function of both capital stocks of the firm; i.e. \( \rho_1 = \rho_1(k_1, k_2) \) and \( \rho_2 = \rho_2(k_1, k_2) \). Firms internalize this effect in their venture posting decisions but capital suppliers do not because they do not know the identity of the firms with which they match at the time of their liquid capital decision. Consequently, the problem of a firm entering the period with capital stocks \( k_1 \) and \( k_2 \) is

\[
J(k_1, k_2) = \max_{v_1, v_2} \left[ f(k_1, k_2) - \rho_1(k_1, k_2)k_1 - \rho_2(k_1, k_2)k_2 - \gamma_1 v_1 - \gamma_1 v_2 + \beta J(k_1', k_2') \right]
\]

\[
s.t. k_i' = (1 - \delta_i)k_i + p(\theta_i)v_i,
\]

for \( i = 1, 2 \). The resulting steady state demand for capital of type \( i \) is

\[
\rho_i(k_1, k_2) = f_{k_i}(k_1, k_2) - \rho_{1,k_i}(k_1, k_2)k_1 - \rho_{2,k_i}(k_1, k_2)k_2 - (r + \delta_i) - \frac{\gamma_i}{p(\theta_i)}. \tag{19}
\]

The terms \( \rho_{1,k_i}(k_1, k_2)k_1 \) and \( \rho_{2,k_i}(k_1, k_2)k_2 \) capture the firm’s strategic reaction to the double holdup problem, as explained below. In turn, the problem of a supplier in market \( i \) entering the period with matched capital \( \kappa_i \) and unmatched liquid capital \( u_i \) is

\[
V(\kappa_i, u_i) = \max_{l_i} \left[ \rho_i(\kappa_i - (l_i - u_i)) + \beta V(\kappa_i', u_i') \right]
\]

\[
s.t. k_i' = (1 - \delta_i)\kappa_i + q(\theta_i)l_i
\]

\[
u_i' = (1 - q(\theta_i))l_i,
\]

for \( i = 1, 2 \). The resulting steady state condition for optimal supply of liquid capital in market \( i \) is

\[
(1 - \beta) = q(\theta_i) \left[ \frac{\rho_i}{r + \delta_i} - \beta \right], \tag{20}
\]

which is the same as condition (8) in the basic model.

The rental rate for capital of type \( i \) that obtains from the continuous pairwise bargaining assumption solves \( \phi_iJ_{k_i}(k_1, k_2) = (1 - \phi_i)[V_{\kappa_i}(\kappa_i, u_i) - V_{u_i}(\kappa_i, u_i)] \), where \( \phi_i \) denotes the bargaining power of the capital supplier in market \( i \). In steady state, this rental rate can be expressed as

\[
\rho_i(k_1, k_2) = \phi_i \left[ f_{k_i}(k_1, k_2) - \rho_{1,k_i}(k_1, k_2)k_1 - \rho_{2,k_i}(k_1, k_2)k_2 \right] + (1 - \phi_i) \left( \frac{r + \delta_i}{1 + r} \right) \tag{21}
\]

for \( i = 1, 2 \). As in the basic model, the rental rate is a weighted average of the firm’s and the supplier’s outside option. The only difference now is that the firm’s outside option of producing with one less unit of \( k_i \) affects rental rates in both markets because a decrease in \( k_i \) affects the marginal productivity of both capital types. As a result, the remaining matched suppliers
from both markets come in to negotiate a new rental rate, the effect of which is captured by $\rho_{1,k_1}(k_1, k_2) k_1$ and $\rho_{2,k_2}(k_1, k_2) k_2$.

Deriving an explicit expression for these two terms involves solving the system of two non-homogenous linear differential equations described by (21). Using spherical coordinate techniques as in Cahuc, Marque and Wasmer [11] yields (see Appendix B.2 for details)

$$\rho_i(k_1, k_2) = \int_0^1 z^{1-\phi_i} f_i(k_1 z^{\chi_{i1}}, k_2 z^{\chi_{i2}}) \, dz + (1 - \phi_i) \frac{r + \delta_i}{1 + r}, \quad (22)$$

where $f_i(k_1 z^{\chi_{i1}}, k_2 z^{\chi_{i2}})$ is defined as the partial derivative of $f(k_1 z^{\chi_{i1}}, k_2 z^{\chi_{i2}})$ with respect to its $i$-th argument; and $\chi_{ij} \equiv 1 - \phi_i / \phi_j$ measures the bargaining power of a capital supplier in market $i$ relative to a capital supplier in market $j$. Similar to the basic model, the rental rate for capital $k_i$ is a weighted average of its infra-marginal productivities. If these inframarginal productivities are a function of both capital stocks, the rental rate depends on both capital stocks.

Given this solution, the firm’s demand for capital $k_i$ in (19) can be expressed similarly to Eq. (11) for the basic model; i.e.

$$\rho_i(k_1, k_2) = OI_i(k_1, k_2) \times f_{k_i}(k_1, k_2) - (r + \delta_i) \frac{\gamma_i}{p(\rho_i)}, \quad (23)$$

only that now, the ‘overinvestment factor’ takes on a more complicated form

$$OI_i(k_1, k_2) = 1 - \frac{1}{f_{k_i}(k_1, k_2)} \int_0^1 z^{1-\phi_i} f_{1i}(k_1 z^{\chi_{i1}}, k_2 z^{\chi_{i2}}) \, dz + \frac{k_2 \int_0^1 z^{1-\phi_i} f_{2i}(k_1 z^{\chi_{i1}}, k_2 z^{\chi_{i2}}) \, dz}{f_{k_i}(k_1, k_2)}. \quad (24)$$

Consider demand for $k_1$ (i.e. $i = 1$). As in the basic model, diminishing marginal productivity ($f_{11} < 0$) implies $OI_1(k_1, k_2) > 1$, providing the firm with a strategic incentive to overaccumulate $k_1$ in order to reduce its holdup problem in the market for $k_1$. At the same time, $k_1$ affects the marginal productivity of $k_2$ and thus the firm’s holdup problem in the market for $k_2$. If the two capital inputs are substitutes (i.e. $f_{12} < 0$), a larger $k_1$ lowers the marginal productivity of $k_2$, thus reinforcing the firm’s strategic incentive to overaccumulate capital. If, by contrast, the two capital inputs are complements (i.e. $f_{12} > 0$), then the firm faces a trade-off because a larger $k_1$ worsens its holdup problem in the market for $k_2$.

The implications for complements can be nicely illustrated for the case of Cobb–Douglas technology $f(k_1, k_2) = k_1^{\alpha_1} k_2^{\alpha_2}$ with $\alpha_1 + \alpha_2 = d < 1$. For this specification, the overinvestment factor in (24) simplifies to

$$OI_i = \frac{1}{1 - \phi_i (1 - \chi_{11} \alpha_1 - \chi_{12} \alpha_2)}.$$  

For equal bargaining powers, $\chi_{11} = \chi_{12} = 1$ and $OI_1 = OI_2 > 1$: the firm has an incentive to overaccumulate capital equally in both markets so as to take advantage of decreasing-returns-to-scale. If $\phi_1 > \phi_2$, then the strategic incentive is stronger for the first type of capital than for the second one (i.e. $OI_1 > OI_2$). Furthermore, if $\phi_1$ is sufficiently large compared to $\phi_2$ such that $1 - \chi_{21} \alpha_1 - \alpha_2 < 0$, the strategic incentive for the second type of capital is reverted; i.e. $OI_2 < 1$. These implications are equivalent to the ones discussed in the labor search context by Cahuc, Marque and Wasmer [11].
To examine the equilibrium effects of the firm’s strategic incentives, combine (19) and (20), respectively, with (22) to eliminate rental rates. After aggregation over firms and capital suppliers, the following solution obtains.

**Proposition 5.** For homogenous production functions of degree \( d < 1 \)

\[
\int_0^1 z^\frac{1}{\phi_i} f_{11}(Rz, z^{X_{12}}) dz \times A_2^D(\theta_2) - \int_0^1 z^\frac{1-\phi_2}{\phi_1} \frac{1}{1-\phi_i} f_{12}(Rz^{X_{21}}, z) dz \times A_1^D(\theta_1^D) < 0
\]

\[\forall(K_1, K_2),\]

there exists a unique decentralized allocation \( \{K_i^D, \theta_i^D\}_{i=1}^2 \) that solves

\[
\theta_i = \frac{1 - \beta}{\gamma_i} \frac{1 - \phi_i}{\phi_i}
\]

(26)

\[
A_i^D(\theta_i^P) \equiv (r + \delta_i) \left( \frac{\gamma_i}{p(\theta_i)(1 - \phi_i)} + \beta \right) = K_i^{d-1} O_i(R_i, 1) f_i(R_i, 1).
\]

(27)

This allocation is always inefficient. In particular:

1. For \( \phi_1 = \epsilon(\theta_1^P) = \phi_2, \theta_1^D = \theta_2^D = \theta_1^P = \theta_2^P = \theta_1^D = \theta_2^D = \theta_1^P = \theta_2^P \) and \( R^D = R^P \) with \( K_1^D > K_1^P, K_2^D > K_2^P \).
2. For \( \phi_1 = \epsilon(\theta_1^P) > \epsilon(\theta_2^P) = \phi_2, \theta_1^D = \theta_2^D = \theta_1^P < \theta_2^P = \theta_1^D = \theta_2^D = \theta_1^P = \theta_2^P \) and
   (a) \( R^D > R^P \) if \( f_{12} > 0 \) with \( K_1^D > K_1^P, K_2^D > K_2^P \) for \( \phi_1 - \phi_2 \) sufficiently small;
   (b) \( R^D < R^P \) and \( K_1^D > K_1^P \) around \( \phi_1 = \phi_2 = \phi \) if \( f_{12} < 0 \) for \( f_{12} \) sufficiently negative
   and \( \phi \) sufficiently large.

**Proof.** Appendix B.2. \( \square \)

The first part of the proposition is analogous to Proposition 4. Condition (26) pins down market tightness \( \theta_1^D \) and \( \theta_2^D \). Given \( \theta_1^D \) and \( \theta_2^D \), condition (27) together with \( R \equiv K_1^D / K_2^D \) then defines a unique solution for \( R^D, K_1^D \) and \( K_2^D \) as long as the two capital inputs are not too substitutable such that condition (25) holds.

The second part of the proposition presents particular examples for which equilibrium over-investment arises. For all examples, market tightness is assumed to be efficient (i.e. \( \phi_1 = \epsilon(\theta_1^P) \) and \( \phi_2 = \epsilon(\theta_2^P) \)) such that \( \theta_1^D = \theta_1^P \) and \( \theta_2^D = \theta_2^P \) so as to isolate the effects of the firm’s strategic incentives from the usual distortionary effects of the two holdup problems.\(^{22}\) First, for the special case in which the efficient match elasticities are the same in the two markets, the equilibrium ratio of the two capital stocks is efficient \( (R^D = R^P) \) but there is overinvestment in each market \( (K_1^D > K_1^P \text{ and } K_2^D > K_2^P) \). As Appendix B.2 shows, this overinvestment result is entirely driven by the assumption of decreasing-returns-to-scale and does not depend on whether the two capital inputs are substitutes or complements. A priori, this result seems to contradict the discussion following Eq. (24), which showed that everything else constant, complementarity between the two capital inputs generally reduces the incentive to overaccumulate capital in the two

\(^{22}\) As the analysis in Appendix B.2 reveals, when \( \phi_1 \neq \epsilon(\theta_1^P) \) and \( \phi_2 \neq \epsilon(\theta_2^P) \), it is not possible to make general statements about equilibrium investment relative to the efficient level because the usual distortionary effects of holdups problems in the two markets interact in complicated ways with the firm’s strategic incentives.
capital markets. In equilibrium, however, firms need to balance the effect of overaccumulating capital in one market with the indirect effect on the other market. When the bargaining powers of the firm in the two markets are the same, the indirect effects cancel each other out and the only remaining strategic incentive is the one from decreasing-returns-to-scale that leads firms to overaccumulate capital.

Once the firm’s bargaining power in the two markets is different but still such that capital market tightness in each market is efficient, the situation becomes more complicated. As Appendix B.2 shows, if the capital factors are complements (i.e. \( f_{12} > 0 \)), \( \phi_1 > \phi_2 \) implies relative overinvestment in the first capital factor; i.e. \( R^D > R^P \); because that is where the firm’s holdup problem is most severe. However, only for \( \phi_1 \) sufficiently close to \( \phi_2 \) is it also the case that the levels of each factor are inefficiently high (i.e. \( K^D_1 > K^P_1, K^D_2 > K^P_2 \)). Intuitively, if the difference between \( \phi_1 \) and \( \phi_2 \) is too large, then the firm’s strategic incentive in the market for \( k_1 \) is sufficiently weighed down by the opposite effect in the market for \( k_2 \) so as to result in inefficiently low absolute levels of capital stocks. This can be nicely illustrated with the above Cobb–Douglas example, which can be shown to imply \((K^D_i / K^P_i)^{1-d} = O_i^{\phi_i} O_j^{1-\alpha_i} \) with \( \lim_{\phi_i \to 1} O_i = 1/\alpha_i \) and \( \lim_{\phi_i \to 1} O_j = 0 \). As \( \phi_i \to 1 \), equilibrium capital stocks \( K^D_1 \) and \( K^D_2 \) therefore both converge to 0; i.e. everything else constant, firms have an incentive to overinvest maximally in the \( i \)-th capital type and nothing in the other capital type. But since the two capital stocks are complements in production, the equilibrium outcome is that investment drops to 0.

By contrast, if the two capital factors are substitutes, then it can be shown that in the neighborhood of \( \phi_1 = \phi_2 = \phi \), \( \phi_1 > \phi_2 \) implies relative underinvestment in the first factor (i.e. \( R^D < R^P \)) as long as the two factors are sufficiently substitutable and the firm’s bargaining power in each market is sufficiently small (i.e. \( \phi \) sufficiently large). For example, if production is CES of the form \( f(k_1, k_2) = (k_1^\sigma + k_2^\sigma)^d/\sigma \) with \( d < \sigma \) (such that \( f_{12} < 0 \)), this condition can be expressed in closed-form around \( \phi_1 = \phi_2 = \phi \) as (see Appendix B.2)

\[
\sigma - d > \frac{1 - \phi}{\phi}.
\]

Intuitively, when the two factors are sufficiently substitutable (i.e. \( \sigma - d \) large) and the holdup problem in each market is important (i.e. \( \phi \) large), then \( \phi_1 > \phi_2 \) implies that the firm substitutes towards the capital factor with the lower bargaining power so as to depress the price of the other factor. Furthermore, by the substitutability of the two factors, \( R^D < R^P \) implies that \( K^D_2 > K^P_2 \); i.e. there is overinvestment in the market where the firm has higher bargaining power.

The analysis confirms that the overinvestment result from the basic model is quite robust. As long as the firm’s bargaining power in each market is such that equilibrium market tightness is close to being efficient, the firm’s strategic incentive to overaccumulate capital can outweigh the usual distortionary effects of holdup problems in at least one of the two markets, thereby leading to equilibrium overinvestment in that market.

### 3.2. Capital and Labor

The environment consists of firms producing with technology \( f(k, n) \), where \( k \) and \( n \) denote capital and labor that are rented from a \([0, 1]\) continuum of atomistic capital suppliers and a \([0, 1]\) continuum of atomistic workers, respectively. This technology is strictly increasing and concave in each of its arguments, satisfies the usual Inada conditions, and exhibits constant-returns-to-scale.
The capital market is subject to exactly the same search friction as in the basic model, characterized by matching function \( m(V_k, L) \) and market tightness \( \theta_k \equiv V_k / L \). The evolution of a firm’s productive capital stock, respectively of a capital supplier’s rented capital stock and unmatched capital is therefore described by Eqs. (1)–(3).

The labor market is modeled as in Smith [51], Cahuc and Wasmer [9,10] and Cahuc, Marque and Wasmer [11]. Workers are infinitely-lived; discount the future at the same rate \( \beta = (1 + r)^{-1} \) as firms and capital suppliers; and have linear preferences for the numeraire good.\(^{23} \)

When employed, workers are paid wage \( w \) that is continuously rebargained with the firm, as will be described below. When unemployed, workers obtain flow value \( b < w \) from non-market activity.

The allocation of workers to firms is subject to a matching friction and occurs in two stages. In the first stage, firms open job vacancies \( v_n \) at flow cost \( \gamma_n \) and search for available workers. Available workers consist of the unemployed from the previous period plus workers who have been newly separated from their jobs.\(^{24} \) Let \( 1 - N \) denote the number of unemployed workers and \( s \) the number of newly separated workers, with \( s \) denoting the exogenous rate at which workers separate from firms. Then \( 1 - N + sN \) defines the number of available workers. Matching between job vacancies and available workers occurs according to matching function \( m(V_n, 1 - N + sN) \), with \( V_n = \int_0^1 v_n \, dt \) denoting the total mass of job vacancies.\(^{25} \) As in the capital market, it is assumed that this matching function exhibits constant-returns-to-scale and satisfies \( \lim_{V_n \to 0} m(V_n, \theta_n) = 1 \) and \( \lim_{V_n \to \infty} m(V_n, \theta_n) = 0 \) with \( \theta_n = V_n / (1 - N + sN) \) denoting labor market tightness. Accordingly, \( p(\theta_n) \equiv m(V_n, 1 - N + sN) / V_n \) and \( q(\theta_n) \equiv m(V_n, 1 - N + sN)/(1 - N + sN) \) denote the probabilities of filling and finding a vacant job, respectively.

In the second stage, newly matched workers join the firm’s existing hires net of separations and produce. After production has taken place, \( sn \) workers separate from the firm and join the unemployed to search for new jobs. Given these assumptions, the evolution of a firm’s employment \( n \) is described by

\[
n' = (1 - s)n + p(\theta_n)v_n. \tag{28}
\]

### 3.2.1. Efficient allocation

The planner’s problem that achieves the (constrained) efficient allocation is

\[
\begin{align*}
P(K, U, N) &= \max_{L, V_k, V_n} \left[ f(K, N) + b(1 - N + sN) - \gamma_k V_k - \gamma_n V_n - (L - U) \\
&\quad + \beta P(K', U', N') \right] \\
\text{s.t. } K' &= (1 - \delta)K + m(V_k, L) \\
U' &= L - m(V_k, L) \\
N' &= (1 - s)N + m(V_n, 1 - N + sN),
\end{align*}
\]

\(^{23} \) It is straightforward to show that none of the results are affected if workers are risk averse.

\(^{24} \) The assumption that separated workers can reenter the labor market in the same period contrasts with the standard Mortensen–Pissarides setup, where separated workers reenter only the following period and thus spend at least one period in unemployment. The assumption has the advantage that zero matching frictions implies zero unemployment (also see Den Haan, Ramey and Watson [17]).

\(^{25} \) Similar to the example with two capital inputs, the same functional form for the two matching functions is used so as to save on notation. None of the below results would be affected if different functional forms were used instead.
where $K = \int_0^1 k \, dt = \int_0^1 \kappa \, d\omega$; $U = \int_0^1 \kappa \, d\omega$ as in the basic model and $N = \int_0^1 n \, dt$. Similar to the model with two capital inputs, the solution to this problem does not necessarily exist for general production functions and, if it exists, is not necessarily unique (see Appendix C.1 for details). Under the assumption of constant-returns-scale technology, however, the following characterization of the equilibrium obtains.

**Proposition 6.** For constant-returns-to-scale technology $f(k, n)$, there exists a unique efficient allocation $\{K^P, N^P, \theta_K^P, \theta_N^P\}$ that solves

$$
\theta_K = \frac{1 - \beta}{\gamma_k} \frac{1 - \epsilon(\theta_K)}{\epsilon(\theta_K)} \\
A^P(\theta_K) \equiv (r + \delta) \left( \frac{\gamma_k}{p(\theta_K)(1 - \epsilon(\theta_K))} + \beta \right) = f_1(R, 1) \\
N = \frac{q(\theta_N)}{s + (1 - s)q(\theta_N)} \\
B^P(\theta_N) \equiv b + \gamma_n \left[ \frac{r + s}{p(\theta_N)} + (1 - s)\epsilon(\theta_N)\theta_N \right] = f_2(R, 1)
$$

with $R \equiv K/N$.

**Proof.** Appendix C.1. 

As in the basic model, condition (29) pins down efficient capital market tightness $\theta_K^P$. Conditions (30)–(31) then jointly determine the efficient levels of $K^P$, $N^P$ and $\theta_N^P$. The remaining efficient allocations $N^P$, $K^P$, $L^P$, $V^P_K$ and $V^P_N$ are computed from the different optimality conditions and constraints evaluated in steady state.\(^{26}\)

### 3.2.2. Decentralized allocation

Analogous to the basic model, the rental rate between firms and matched capital suppliers is determined ex-post through continuous pairwise bargaining. Similarly, and following the assumptions in Stole and Zwiebel [52,53], Smith [51], Cahuc and Wasmer [9,10] and Cahuc, Marque and Wasmer [11], firms and workers are subject to non-binding ‘employment-at-will’ contracts, with wage rates determined via ex-post pairwise bargaining at any time before production starts. Analogous to the previous example with two capital inputs, as long as capital and labor are not perfect substitutes, the resulting rental rate and wage rate are functions of the firm’s capital stock $k$ and its employment $n$; i.e., $\rho = \rho(k, n)$ and $w = w(k, n)$. Firms internalize this effect in their optimal investment and hiring decisions. By contrast, suppliers take $\rho$ as given because they do not know the identity of the firm at the time of their liquid capital decision. Likewise, workers consider $w$ as exogenous because they not take any optimal decisions (if not employed, the automatically search at no cost and if employed, they supply one unit of labor).\(^{27}\)

\(^{26}\) As emphasized in the beginning of the section, the definition of equilibrium would hold for more general homogenous production functions as long as $k$ and $n$ are not too substitutable (for constant-returns-to-scale they are necessarily complements). The derivations for this more general case are essentially equivalent to the ones for the two-capital case analyzed above.

\(^{27}\) Equivalently, workers do not form an ex-ante coalition to participate in the labor market as an optimizing union. See Stole and Zwiebel [52] for discussion.
The problem of a firm entering the period with capital stock $k$ and employment $n$ is

\[
J(k,n) = \max_{\nu_k, \nu_n} \left[ f(k,n) - \rho(k,n)k - w(k,n)n - \gamma_k \nu_k - \gamma_n \nu_n + \beta J(k',n') \right]
\]

s.t. $k' = (1-\delta)k + p(\theta_k)\nu_k$

\[ n' = (1-s)n + p(\theta_n)\nu_n. \]

The resulting steady state demands for new capital and new workers are, respectively,

\[
\rho(k,n) = f_k(k,n) - \rho_k(k,n)k - w_k(k,n)n - (r+\delta)\frac{\gamma_k}{p(\theta_k)}
\]

(33)

\[
w(k,n) = f_n(k,n) - \rho_n(k,n)k - w_n(k,n)n - (r+s)\frac{\gamma_n}{p(\theta_n)}.
\]

(34)

The terms $\rho_k(k,n)k$ and $w_k(k,n)n$ in (33) embody the firm’s strategic incentive with respect to capital accumulation; i.e. the firm internalizes that an additional unit of capital affects rental rate negotiations with suppliers but also influences wage negotiations with workers. The terms $\rho_n(k,n)k$ and $w_n(k,n)n$ in (34) capture the equivalent strategic incentives with respect to hiring.

The problem of a capital supplier is exactly identical to the one in the basic model and not repeated here (see Appendix C.2 for details). The resulting optimal liquid capital supply condition is

\[
(1-\beta) = q(\theta_k) \left[ \frac{\rho}{r+\delta} - \beta \right].
\]

(35)

The value of a matched (employed) worker is

\[
E = w + (1-s(1-q(\theta_n)))\beta E' + s(1-q(\theta_n)))S',
\]

and the value of an unmatched (i.e. searching) worker is

\[
S = b + q(\theta_n)\beta E' + (1-q(\theta_n))\beta S'.
\]

The rental rate and the wage rate that come out of the continuous pairwise bargaining with alternating-offer protocol solve $\phi_k J_k(k,n) = (1-\phi_k)[V_k(k,u) - V_u(k,u)]$ and $\phi_n J_n(k,n) = (1-\phi_n)(E - S)$, respectively, with $\phi_k$ and $\phi_n$ denoting the bargaining power of capital suppliers and workers. After some rearrangement, this yields the following steady state expressions for the rental rate and the wage rate

\[
\rho(k,n) = \phi_k \left[ f_k(k,n) - \rho_k(k,n)k - w_k(k,n)n \right] + (1-\phi_k) \frac{r+\delta}{1+r}
\]

(36)

\[
w(k,n) = \phi_n \left[ f_n(k,n) - \rho_n(k,n)k - w_n(k,n)n \right] + (1-\phi_n)w_R,
\]

(37)

where $w_R \equiv b + \frac{\phi_n}{1-\phi_n} \gamma_n (1-s)\theta_n$ is the worker’s reservation wage. Together, (36) and (37) form a system of non-homogenous linear differential equations of $\rho$ and $w$ in $k$ and $n$. Application of the spherical coordinate solution techniques of Cahuc, Marque and Wasmer [11] yields

\[
\rho(k,n) = \int_0^1 \frac{1}{z^{\phi_k}} f_1(kz, nz^{\chi_k}) dz + (1-\phi_k) \frac{r+\delta}{1+r}
\]

(38)

\[
w(k,n) = \int_0^1 \frac{1}{z^{\phi_n}} f_2(kz^{\chi_n}, nz) dz + (1-\phi_n)w_R
\]

(39)
with $\chi_{kn} \equiv \frac{1-\phi_k}{\phi_n-\phi_k}$ and $\chi_{nk} \equiv 1/\chi_{kn}$. As in the basic model, the price of capital is a weighted average of infra-marginal productivities with respect to capital. Similarly, the wage rate is a weighted average of infra-marginal productivities with respect to labor. Using these solutions, the capital and labor demands in (33) and (34) can be expressed as

$$
\rho(k,n) = OI(k,n) \times f_k(k,n) - (r+\delta) \frac{\gamma_k}{p(\theta_K)} 
$$

$$
w(k,n) = OE(k,n) \times f_n(k,n) - (r+s) \frac{\gamma_n}{p(\theta_N)}
$$

with the overinvestment factor $OI(k,n)$ and the overemployment factor $OE(k,n)$ being defined as

$$
OI(k,n) \equiv 1 - k \int_0^1 z^{1-\phi_k} f_{11}(kz, nz^{\chi_{kn}}) dz + n \int_0^1 z^{1-\phi_n} f_{21}(kz^{\chi_{nk}}, nz) dz
$$

$$
OE(k,n) \equiv 1 - k \int_0^1 z^{1-\phi_k} f_{12}(kz, nz^{\chi_{kn}}) dz + n \int_0^1 z^{1-\phi_n} f_{22}(kz^{\chi_{nk}}, nz) dz
$$

As before, these two definitions closely resemble the definition of the overemployment factor in Cahuc, Marque and Wasmer [11] and so does their interpretation. Consider first the over-investment factor $OI(k,n)$. As in the basic model, decreasing marginal productivity of capital (i.e. $f_{11} < 0$) provides the firm with an incentive to overaccumulate capital in order to reduce its holdup problem in the capital market. At the same time, the firm needs to take into account that capital affects the marginal productivity of labor. Since capital and labor are complements (i.e. $f_{12} > 0$) when production is constant-returns-to-scale, a higher capital stock worsens the firm’s holdup problem in the labor market, thus reducing the overinvestment motive of the firm. Exactly the same considerations apply to the firm’s hiring decision. One the one hand, larger employment reduces the marginal productivity of labor (i.e. $f_{22} < 0$) and therefore the wage rate. On the other hand, larger employment increases the marginal productivity of capital (i.e. $f_{21} > 0$) and therefore the rental rate. In a constant-returns-to-scale environment, the firm therefore faces a trade-off that, depending on the details of technology and the bargaining powers of capital suppliers and workers, may result in the firm having a strategic incentive to overaccumulate capital (i.e. $OI(k,n) > 1$), to overhire (i.e. $OE(k,n) > 1$), or both.\(^{28}\)

To compute the equilibrium allocation, combine capital supply in (35) and capital demand in (40), respectively, with the rental rate in (38); and the wage rate in (39) with labor demand in (41). After aggregation over firms and capital suppliers, the two equations lead to the following characterization of the decentralized allocation.

**Proposition 7.** For constant-returns-to-scale production function $f(k,n)$, there exists a unique decentralized allocation $\{K^D, N^D, \theta^D_K, \theta^D_N\}$ that solves

\(^{28}\) The above analysis could be easily extended to a multi-worker setting as is done in Cahuc, Marque and Wasmer [11] in the context of a perfectly competitive capital market. Their derivations show that if capital is substitutable to a type of labor with strong bargaining power (e.g. relatively low-skilled but highly unionized labor), then this provides incentive for the firm to overaccumulate capital. This incentive is reinforced if the capital market itself is subject to a holdup problem, as is the case in the present context. The paper refrains from pursuing this multi-worker case further since even for very simple production functions, it is impossible to establish existence and uniqueness of the equilibrium.
\[
\theta_K = \frac{1 - \beta}{\gamma_k} \frac{1 - \phi_k}{\phi_k}
\]

(44)

\[
A^D(\theta_K) \equiv (r + \delta) \left( \frac{\gamma}{p(\theta_K)(1 - \phi)} + \beta \right) = OI(R, 1) \times f_1(R, 1)
\]

(45)

\[
N = \frac{q(\theta_N)}{s + (1 - s)q(\theta_N)}
\]

(46)

\[
B^D(\theta_N) \equiv b + \frac{\gamma_n}{1 - \phi_n} \left[ \frac{r + s}{p(\theta_N)} + (1 - s)\phi_n\theta_N \right] = OE(R, 1) \times f_2(R, 1).
\]

(47)

This allocation is generally inefficient except for one special case. In particular:

1. For \( \phi_k = \epsilon(\theta_P^K) = \epsilon(\theta_P^N) = \phi_n \), \( \theta_P^D_K = \theta_P^N \) and \( RD = RP, KD = KP \) and \( ND = NP \); i.e. the decentralized allocation is efficient.

2. For \( \phi_k = \epsilon(\theta_P^K) > \epsilon(\theta_P^N) = \phi_n \), \( RD > RP \) and \( KD > KP \) and \( ND > NP \) as long as \( \phi_k \) and \( \phi_n \) are sufficiently close together.

Proof. Appendix C.2. □

The capital market part of the equilibrium is analogous to the basic model: Eq. (44) pins down capital market tightness \( \theta_P^D_K \); and Eq. (45) pins down the equilibrium capital–labor ratio \( R^D \). Given this ratio, Eqs. (46) and (47) then jointly determine labor market tightness \( \theta_P^D_N \) and employment \( ND \); and the equilibrium values of the other variables can be inferred using the different constraints and definitions.

As in the two-capital example above, the second part of the proposition considers only situations for which the bargaining powers \( \phi_k \) and \( \phi_n \) are such that market tightness is efficient; i.e. \( \theta_P^D_K = \theta_P^N \) and \( \theta_P^D_N = \theta_P^N \). This isolates the equilibrium implications of the firm’s strategic incentives from the usual distortionary effects of holdup problems. Then, for the special case of \( \phi_k = \epsilon(\theta_P^K) = \epsilon(\theta_P^N) = \phi_n \), the decentralized allocation is efficient. Intuitively, efficiency occurs because under constant-return-to-scale, the firm’s incentive to overaccumulate capital is exactly offset by the firm’s incentive to overhire. This knife-edge case illustrates that multiple holdup problems do not necessarily exacerbate each other as is often the case in the literature (e.g. Aruoba, Waller and Wright [4]). Rather, if the factors subject to holdup problems are complementary in the firms’ objective, then depending on the relative bargaining powers of the different parties involved, the decentralized economy may be relatively close to efficiency even if each holdup problem on its own severely distorts the firm’s incentives.

Once bargaining powers of suppliers and workers are different (but still such that market tightness is efficient), the capital–labor ratio departs from the efficient value. In particular, if \( \phi_k > \phi_n \), the firm accumulates relatively more capital than labor because the firm’s holdup problem in the capital market is worse than in the labor market. If the difference between \( \phi_k \) and \( \phi_n \) is too large, then the distortions arising from the firm’s strategic incentives can become so large as to result in inefficiently low levels of either capital or employment or both. As for the two-capital case, this result can be illustrated nicely for Cobb–Douglas production (see Appendix C.2).

\[29\] It is straightforward to show that under decreasing-returns-to-scale, the result from the two-capital example would arise: given equal bargaining powers, the firm would both overinvest and overemploy so as to lower productivity and with it the rental rate and the wage rate.
The main point of this exercise remains that there exists a range of bargaining powers for which the firm’s strategic incentives from holdup problems in capital market outweighs other distortions from holdup problems that would, on their own, lead to underinvestment. As a result, the economy overinvests.

4. Conclusion

This paper shows that two characteristics of many decentralized capital markets – specificity and absence of ex-ante binding contracts – lead to a potentially important holdup problem that provides the firm with a strategic incentive to overaccumulate capital. In equilibrium, this strategic incentive can outweigh the usual distortions from holdup problems that would, on their own, lead to underinvest. As a result, the economy overinvests.

This overinvestment result contrasts with much of the literature and offers a cautionary tale that holdup problems do not necessarily imply underinvestment. Instead, the paper shows that it all depends on the specifics the production function and the relative importance of the firm’s holdup problem in each market.

Whether in reality, the firm’s strategic incentive leading to overinvestment outweighs the usual distortions leading to underinvestment is a quantitative question that exceeds the scope of this paper. Yet, the policy implications of the overinvestment channel are clearly important. For example, overinvestment provides a rationale for capital income taxation or, at the least, counteracts other forces that, on their own, imply underinvestment and capital income subsidies.

The overinvestment result may also help explain a number of empirical phenomena; e.g. why we observe very large firms in capital-intensive industries or why firms display lumpy investment behavior. Moreover, Caballero and Hammour [8] argue that worsening holdup problems in labor markets and factor substitutability provide an explanation for the sustained increase of the capital–labor ratio in European countries from the 1970s through the 1990s. The analysis in Section 3 suggests that holdup problems in capital markets and the associated strategic incentive of the firm to overaccumulate capital amplifies these effects. Finally, the misallocation of resources implied by the firm’s strategic incentive in capital markets with holdup problems may provide another source for persistent productivity differences across countries.

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30 De Meza and Lockwood [15,16] explore alternative mechanisms due to coordination failure and heterogeneity across agents that imply overinvestment as a result of holdup problems. Outside the holdup literature, Chien and Lee [12] or Lagos and Rocheteau [39] present other mechanism that can lead to overinvestment. Furthermore, in the industrial organizations literature, strategic behavior by incumbents in imperfectly competitive markets can generate overinvestment. See Tirole [54] for a review.
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Appendix. Supplementary material

All the proofs of the results presented in this paper are available in an on-line appendix at http://dx.doi.org/10.1016/j.jet.2014.02.004.

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